Theory of Sato-Fourier
Hyperfunctions IV

English Edition

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Part IV

The realizations of (vector valued) Sato-Fourier hyperfunctions as boundary values of (vector valued) holomorphic functions or slowly increasing holomorphic functions
Introduction

This book is the fourth part of this series of the theories of Sato-Fourier hyperfunctions.

In this book, we realize the (vector valued) Sato-Fourier hyperfunctions as boundary values of several kinds of (vector valued) holomorphic functions respectively.

In each case, we prove The Dolbeault-Grothendick resolution, The Oka-Cartan-Kawai Theorem B, The Malgrange’s Theorem, The Serre’s duality theorem and the Martineau-Harvey’s Theorem. Thereby, we prove the Sato’s Main Theorem.

As for these Theorems, we remark the following. Several types of the Oka-Cartan-Kawai Theorem B are the generalizations of the theorems of Oka [49], Cartan [2], Kawai [36], [37] and Hörmander [7]. The Malgrange’s Theorems are the generalizations of Malgrange [43]. The Serre’s duality Theorems are the generalizations of Serre [57]. The Martineau-Harvey’s Theorems are the generalizations of Martineau [44] and Harvey [6]. The Sato’s Theorems are the generalizations of Sato [54], [55].

For the realizations of Sato-Fourier hyperfunctions, we use the following two methods (1) and (2):

(1) The duality method. By using this method, we define the Sato-Fourier hyperfunctions as several kinds of the classes of analytic functionals and the vector valued Sato-Fourier hyperfunctions as several kinds of the classes of analytic linear mappings.

(2) The algebro-analytic method. By using this method, we realize the Sato-Fourier hyperfunctions as several kinds of the boundary values of holomorphic functions and the vector valued Sato-Fourier
hyperfunctions as several kinds of the boundary values of vector valued holomorphic functions.

The two kinds of realizations are constructed independently and they are equivalent.

Here I show my heartfelt gratitude to my wife Mutuko for her help of typesetting this manuscript.

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